A STUDY OF MISCONCEPTIONS ABOUT GEOMETRY IN MIDDLE SCHOOL LEARNERS

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Abstract

The main purpose of this research is to study the misconceptions about geometry in middle school learners. Quantitative research methodology was mainly used to find out students' misconceptions in geometry. The design adopted in this study was a descriptive research design. This study was conducted in Yangon Region. There are four districts in Yangon Region: East, West, South and North. One township from each district was randomly selected. Two basic education high schools and one basic education middle school from each township were randomly selected. Thus, eight high schools and four middle schools were included in this study. To obtain the required data, (600) students and (68) mathematics teachers were participated in this study. Two instruments: a test for Grade Eight students' misconceptions in geometry and a questionnaire for mathematics teachers' perceptions were employed. The test included eight content areas: angles, triangles, congruence of triangles, quadrilaterals, parallel lines, circles, Pythagoras' theorem, and areas and volumes. The questionnaire consisted of (15) items on a five point Likert scale of (1) to (5) to explore attitude, attention, participation, doing exercises and conceptual understanding. The internal consistency (Cronbach's Alpha) for the test and the questionnaire were (.740) and (.866) respectively. The research findings revealed that most of the students had misconceptions in geometry, and least of the students had understanding the concept and less understanding the concept in geometry. For Grade Eight students, the highest level of misconceptions in geometry was in Pythagoras' theorem and the lowest level was in angles. For Grade Eight students, the highest level of the causes of misconceptions in geometry was in doing exercises (rote learning) and the lowest level was in attitude. To reduce misconceptions in geometry, it is necessary to ensure meaningful learning and quality education.

Keywords: geometry, misconception.

Introduction

Competency in mathematics is a necessity in the modern world because it not only plays a vital role in supporting the education system but also is a practical discipline reaching into a wide variety of fields such as science, technology, etc. Similarly, geometry, an area of mathematics, helps in developing good reasoning and it is applicable to solve human and natural problems like everyday life problems. Thus, it is necessary to be successful teaching and learning in geometry. According to National Council of Teachers of Mathematics (NCTM) (2000), students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge. In learning geometry, if learners have misconceptions before teaching, these can lead to confusion, frustration, errors and prevent learners to build up confidence and positive attitude towards mathematics learning, to value mathematics and to appreciate the beauty of mathematics. Therefore, in order to prevent misconceptions in geometry, there is necessary to understand why these misconceptions emerge and persist.

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Background of the Study

Learners' thinking about geometry is critical in learning geometry. Although geometric concepts have a visual aspect, learners consider them difficult to learn (NCTM, 1989, cited in Kembitzky, 2009). Clements and Battista (1992, cited in Pusey & Lousie, 2003) indicated the reasons for learners' misconceptions about geometric concepts as follows: learners do not understand subjects sufficiently, they overgeneralize specific rules about geometric expressions, they mostly learn by rote, and they cannot understand concepts exactly. Besides, according to National Education Strategic Plan (NESP) (2016), the current emphasizes on rote memorization of factual information in learning and assessment. This will make learners' misconceptions.

Some fundamental misconceptions often originate from earliest years of schooling, but may persist at higher levels. Besides, learners can enter into a classroom having misconceptions that have the potential to derail new learning. This can justify why it is important to carry out a research on the misconceptions held by learners in the teaching and learning of concepts.

If a learner has a misconception prior to learning a subject, this may prevent him/her from learning the new subject properly, thereby leading to new misconceptions. Thus, it is necessary to know learners' common misconceptions to reduce learners' conceptual difficulties and to become successful learning.

Statement of the Problem

In Myanmar, middle school level is the bridge between the primary school level and the high school level. Students' meaningful learning of geometry could help them solve and appreciate real-life problems. But, many students have some difficulties in geometry.

They thought themselves that "They cannot do geometry. Geometry and they are like oil and water." They mostly learn rote learning. They have accustomed to learning through memorization with hard work, but not independent thinking and creativity. Then, they gradually become math phobia, especially in geometry.

Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well (National Council of Teachers of Mathematics, NCTM, 2000, cited in Kembitzky, 2009). Identification of students' specific misconceptions is especially important for students with learning disabilities and low performing students (Salvia & Ysseldyke, 2004, cited in Zuya & Kwalat, 2015). By pinpointing students' misconceptions, teacher can provide instruction targeted to the student's area of need. Therefore, this study is aimed to study students' misconceptions in geometry.

Purposes of the Study

The main purpose of this research is to study the misconceptions about geometry in middle school learners.

The specific purposes are as follows:

1. To find out students' misconceptions in geometry
2. To investigate the mathematics teachers' perceptions on the causes of students' misconceptions in geometry
3. To give suggestions for reducing misconceptions in geometry at the middle school level

Research Questions

The research questions are as follows:

1. To what extent do Grade Eight students have misconceptions in geometry?
2. In which content area do Grade Eight students have the highest level of misconceptions in geometry?
3. In which content area do Grade Eight students have the lowest level of misconceptions in geometry?
4. In which cause do Grade Eight students have the highest level of misconceptions in geometry?
5. In which cause do Grade Eight students have the lowest level of misconceptions in geometry?

Scope of the Study

This research has its own particular limitations. The first limitation is related to the fact that the participants of the study came from only twelve selected schools from Yangon Region. Participants in this study are Grade Eight students from the twelve selected schools in four districts (East, West, South and North) in the academic year (2018-2019). The second limitation is the content areas of the subject. The content areas are limited to eight areas such as angles, triangles, congruence of triangles, quadrilaterals, parallel lines, circles, Pythagoras' theorem and areas and volumes, based on mathematics textbooks volume II (Grade Six, Grade Seven and Grade Eight).

The third limitation is that this study is only concerned with three levels of misconceptions (recognition/visualization, analysis, informal deduction) according to the van Hiele theory. Besides, in this study, other ways that foster learners' misconceptions in geometry at the middle school level were not tried to find out. The fourth limitation is that the questionnaire for mathematics teachers' perceptions was only used to investigate the mathematics teachers' perceptions on the causes of students' misconceptions in geometry.

Definition of Key Terms

Geometry: Geometry is a branch of mathematics concerned with point, straight line, plane figures, space, spatial figures, the relations between them and the measures of geometric figures including length, angle, area and volume, etc. (Biber, Tuna & Korkmaz, 2013, cited in Zuya & Kwalat, 2015).

Misconception: A misconception is a concept that is not in agreement or is different from the accepted understanding of in a field and that are presumed to interfere with the acquisition of new knowledge (Resnick, 1983, cited in Mestre, 1989).

Significance of the Study

Geometry is the study of shapes, their relationships and their properties (Bassarear, 2012, cited in Luneta, 2015). In teaching geometry, learning concepts with understanding is essential. When learners have conceptual knowledge, they know more than isolated facts and methods. They are able to represent mathematical situations in different ways and know how different representations can be useful for different purposes.

According to Hibert (1986), conceptual knowledge is achieved in two ways: by the construction of relationships between pieces of information and by the creation of relationships between existing knowledge and new information that is just entering the system. However, learners' rote (and frequently faulty) knowledge often interferes with their informal (and usually correct knowledge). This may lead to misconceptions. If learners have misconceptions, their learning becomes more challenging. If a learner holds a misconception, it distorts correct concepts.

An understanding of common students' misconceptions, and effective strategies to help students avoid them, is an important aspect of mathematical pedagogical content knowledge (Graeber, 1999). If teachers "un-teach" or "undo" students' misconceptions first, students cannot become successful learners with positive outcomes.

With a greater understanding of students' misconceptions of the skills that are prerequisites to the learning of new material in basic mathematics, teachers may be able to improve student learning. To do so, teachers must understand students' misconceptions. Therefore, to help teachers, this study is intended to find Grade Eight students' misconceptions in geometry.

Review of Related Literature

Mathematics Education

Mathematics plays a vital role in the day to day life. The knowledge of fundamental process of mathematics and the skill to use them are the preliminary requirements of human beings in any society of modern time. Thus, it is a very important subject.

Mathematics is necessary for the development of scientific, technical, monetary and commercial activities around the life of an individual and the community. Mathematics aids in understanding other subjects, especially science subjects, and in teaching mathematics. Therefore, it is necessary to enable the learner develop clear and logical thinking needed for analysis of both academic and everyday life situation.

The Reasons for Learners' Failing in Mathematics

According to Lieback (n.d.), some learners failed at mathematics because of rote learning, anxiety, understanding and attitude.

Rote Learning

Learning by heart is the memorization of information based on repetition. Some consider rote learning to be a necessary step in learning certain subjects. Although there are many advantages, rote learning can lead to many disadvantages. Rote work is widely considered an inaccurate representation of the learner's intelligence and comfort with the subject matter. Because it relies so heavily on memory, and does not necessarily reflect the student's core understanding of the subject, theory and memorization can portray a false sense of accomplishment.

Anxiety

Anxiety is possible to become over-motivated. Many psychologists have shown that high anxiety impedes learning. Learners with negative attitude towards geometry have performance problems simply because of anxiety.

Understanding

When concepts are explained, students have to rely on their understanding and imagination to figure out what is being taught. If not, learning will not be a continuing process.

Attitude

Attitude plays an important role on leaners' geometry achievement. Learners' interest in geometry is associated with their achievement in geometry. In addition, learners' attitude towards geometry had a direct effect on their achievement. In spite of the recognition given to geometry among mathematics topics, it is evident that learners still show negative attitude towards geometry thereby leading to poor performance.

The above causes can make learners fail in mathematics as well as geometry and also lead to some misconceptions. Therefore, teachers should try to reduce learners' anxiety and ensure meaningful learning, conceptual understanding and positive attitude.

The Teaching of Geometry

Geometry is the branch of mathematics that deals with space, figures in space and with properties of those figures such as size and shape. It involves the great importance place in people's lives to fulfill the need of human beings to specify quantities, to measure figures, land and earth, and make maps. Thus, it is a very important subject in daily life.

Geometric representations can be used to help students make sense of other areas of mathematics: fractions and multiplication in arithmetic, the relationships between the graphs of functions (of both two and three variables), and graphical representations of data in statistics. It is also an important branch of mathematics and it is well known to be the one of basic skills to be mastered. Therefore, it is an important area in the school mathematics curriculum and necessary to teach.

The Objectives of Teaching Geometry at the Middle School Level

According to Sidhu (1995), the objectives of teaching geometry at the middle school level are as follows:

1. To familiarize with the use of the straight edge, protractor, compass, an set-square and to draw the simple geometric figures with them
2. To help students learn the important facts, relations and principles of geometric figures by drawing, measuring, comparing, experimenting, discussing, explaining, questioning, conjecturing, verifying and testing
3. To help students learn the geometric vocabularies
4. To develop an understanding of the inductive method as a way of looking for geometric facts, relations and principles
5. To acquaint the students with the characteristics of good geometric notation

Different Stages of Teaching Geometry

According to Sidhu (1995), there are three stages of teaching geometry: the practical stage, the stage of reasoning and the systematizing stage.

1. The Practical Stage

The practical stage is the stage of the geometry of the classroom and environment. It is the period of experimentation, observation, recognition and construction. In the practical stage, students will be expected to acquaint themselves with the common geometrical concepts and figures. By examining and handling geometric models, straight lines, curved lines, angles, triangles, polygons, circles, cubes, cuboids, cylinders, sphere, etc., students will be mainly guided to experience the symmetry, variety, regularity and beauty of forms in the nature and practical arts. Besides, students will be taught how to keep and handle the instruments. The work will center round the observing and drawing of common geometric figures. But it should not be taken to mean that practical geometry ends with this stage. It is the beginning of the entire geometrical work.

2. The Stage of Reasoning

The stage of reasoning is the stage to learn to prove theorems and exercises. The proofs will have to be presented in both the practical and theoretical forms to provide flawless understanding. In this stage, students will be expected to get used to reasoning without dependence on real and concrete instances. The reasoning will more and more incline to the side of argument than stick to the obvious nature of observation. Informal reasoning will be encouraged and made interesting and attractive at this stage. Students will be enabled to know the interesting theorems of plane geometry and to solve easy riders. Students will be prepared for the more formal reasoning to come in the next stage.

3. The Systematizing Stage

The systematizing stage is the stage of acquisition of mastery in reasoning. The reasoning will be more rigorous but properly suited to the mental age of the students. Practice in logical reasoning will be more important than convincing them that the facts are true. Dependence on axioms will also be reduced.

Importance of Conceptual Understanding in Geometry

Geometry is a way of thinking. Geometric understanding builds its foundation on geometric meaning. When a student understands the meaning underlying principles of geometric concepts, he or she has conceptual knowledge in geometry. Conceptual understanding refers to an integrated and functional grasp of geometric ideas. Conceptual understanding knows more than isolated facts and methods.

In the 21^{st} century, students need to develop conceptual understanding in order to flourish and solve problems as adults in the present changing environment. The successful student understands the ideas and has the ability to transfer their knowledge in new situations and apply it to new contexts. Therefore, conceptual understanding is an important component and teachers should help learners ensure conceptual understanding.

Before learning the new concept, if learners have some misconceptions, it may disturb to understand the concept and use the concept appropriately. Hence, learners will also encounter

conceptual difficulties and many challenges. That is why, to ensure conceptual understanding, it is necessary to find out and understand learners' misconceptions.

Nature of Misconception

Students' thinking consists of many things. Formulae, relevance, tedium and enjoyment are part of their attitudes and thinking about mathematics. Many students do not come to the classroom as "blank slates". Rather they come with informal theories constructed from everyday experiences. These theories have been actively constructed. They provide an everyday functionality to make sense of the world but are often incomplete half-truths (Mestre, 1989). So, they become misconceptions.

Students learn concepts, and sometimes they can also learn misconceptions – in spite of whatever teachers try to teach them. Some errors or mistakes are persistent because of misconceptions. However, although misconceptions are consciously made, mistakes are usually due to carelessness. They cannot do the same for misconceptions. Misconceptions are committed because students think they are correct.

Students often approach learning situations with misconceptions or with prior knowledge that actually impedes learning. During experiences with a concept or a process (or a procedure), a student focuses on whatever the experiences appear to have in common and connects that information to information already known. Students have done the right beginning to solve the problems but they couldn't reach correct answer because of some mathematical misconception.

Some students will make some generalizations that are not correct and many of these misconceptions will remain hidden unless the teacher makes specific efforts to uncover them. Misconceptions are a problem for two reasons. First, students tend to be emotionally and intellectually attached to their misconceptions, partly because they have actively constructed them and partly because they give ready methods for solving various problems. Second, they definitely interfere with learning when students use them to interpret new experiences.

Another problem that leads to very serious learning difficulties in mathematics is those misconceptions that student may get from previous inadequate teaching, informal thinking, or poor remembrance. Therefore, it is very important to recognize student misconceptions and to re-educate students to correct mathematical thinking.

Students' Misconceptions in Geometry

Knowledge is not transferred from person to person. The individual does not passively receive knowledge from the environment, but is an active participant in the construction of his/her own knowledge. The construction activity involves the reception of new ideas and the interaction of these with the students' existing ideas. Student errors are the result or the product of previous experience in the mathematics classroom.

Confusions or misconceptions that occur during the fundamental learning of underlying concepts, may lead to greater difficulties throughout school and beyond. It is important that students develop correct concepts. Misconceptions are conceptual or reasoning difficulties that hinder students' mastery of any discipline.

Geometric misconceptions are the students' unacceptable explanations and responses about geometric concepts as a result of passing through inaccurate, incorrect and confused life, and learning experiences which contradict with the approved geometric concepts by mathematics

teaching professionals partially or wholly. This is based on the fact that students' awareness of such concepts is in contrast with those who are specialized in these concepts.

Many students have problem in comprehending geometric concepts, which is an important aspect of learning mathematics. Reasons for students' misconceptions in geometry include students' reliance on the physical appearances of the figures, inability to associate geometric properties with one another, overgeneralization and rote learning (Ozerem, 2012).

Some common misconceptions in geometry among students are as follows. Clements and Battista (1992, cited in Pusey & Lousie, 2003) said geometric shapes presented in non-standard forms are hardly recognized by many students, as they perceive a square as not a square if it is not on a horizontal base. Furthermore, many students have problems in perceiving class inclusions of shapes, for example, they do not think that a square is a rectangle, or a square is a rhombus, and a rectangle is a parallelogram.

Other common misconceptions include, using the bottom line as the base of the triangle in calculating the area of a triangle; larger space means larger angle; inability to understand the angles in parallel lines – alternate and corresponding angles; inability to recognize and perceive the properties of quadrilaterals; learning formulas and definitions inadequately.

Many teachers have observed that many students have numerous misconceptions about geometry when a teacher discusses a geometry proof problem in class, it generally involves oral presentation of a formal proof and body movements pointing at different parts of the figure of the problem. Students must watch, listen, jot notes, and think as a lecture proceeds. They have to refer to many elements of the instruction and incorporate them into their memory (Sweller, 1988, cited in Ozerem, 2012). This often causes cognitive overload and poses a negative effect on students' learning.

Addressing the difficulties and misconceptions in learning geometry, Duval, Healy and Hoyles (1998, cited in Ozerem, 2012) explained that geometry instruction is often more complex than that of numerical operations or elementary algebra. It is therefore more important that geometry instructions must incorporate new and tested approaches such as using visual and multimedia tools in the classroom. Furthermore, to be effective teaching learning in geometry, teachers should try to apply appropriate theories, approaches and methods according to their students' previous knowledge.

The van Hiele Theory

According to Senk (1989, cited in Makhubele, 2014), the van Hiele theory of levels of thought in geometry is the most famous and prominent model used in the teaching of geometry. The two Dutch mathematics educators, Pierre van Hiele and his wife Dina van Hiele investigated the role of instruction in assisting learners to acquire geometric knowledge and raise their thought levels.

According to van Hiele (1986, cited in Makhubele, 2014), there are five levels in students' geometric understanding. They are:

1. **Level 0 (recognition/visualization):** Learners identify, name, compare and operate on geometric figures on the basis of their appearance in a holistic manner at this level they are able to recognize and name figures based on the characteristics of the figure. The emphasis at this level is on the shapes that students can observe, feel, build, take part, or work with in some manner.

2. **Level 1 (analysis):** Learners analyze figures in terms of their components and discover the relationships among those components as well as derive the properties/rules of a class of shapes empirically. They are able to consider all shapes within a class rather than a single shape. Learners begin to appreciate that a collection of shapes goes together because of properties.
3. **Level 2 (informal deduction):** Learners are able to interrelate logically previously discovered properties/rules by giving or following informal arguments. They begin to be able to think about properties of geometric objects.
4. **Level 3 (deduction):** Learners are able to prove theorems deductively and establish interrelationships among networks of theorems. Learners begin to appreciate the need for a system of logic. They are able to work with abstract statements about geometric properties and make conclusions based on logic than intuition. They also prove theorems using clearly articulated logical reasoning.
5. **Level 4 (rigor):** Learners establish theorems in different postulation systems and analyze/compare these systems. At this level, there is an appreciation of the distinctions and relationships between different axiomatic systems. Learners can understand the necessity of precision and the interrelationships between mathematical systems or structures. This means learners can reason abstractly without any reference to a concrete model.

Among them, the first three levels (recognition/visualization, analysis and informal deduction) will be examined in this study.

Research Method

This study is concerned with the misconceptions about geometry in middle school learners. The purpose of this study is to find out the students' misconceptions in geometry at the middle school level. Research design and procedure, instruments, population and sample, and data analysis are presented to address the research questions.

Research Design and Procedure

Research design and procedure are presented as follows:

Research Design

The research design for this study was a descriptive research design. In this study, data were mainly collected through a quantitative method. The researcher used descriptive research design to collect data about the Grade Eight students' misconceptions in geometry. Quantitative descriptive research data were mainly collected from the test and the questionnaire.

Procedure

To obtain the required data, the researcher had done the following steps in this study.

Step 1: Formulating the Problem

The students' misconceptions in learning geometry are still in the unsatisfied condition. So, to study the misconceptions about geometry in middle school learners is necessary.

Step 2: Compiling Related Literature

The researcher sought out the literature related to this study through reading books. Moreover, the researcher studied the literature from the Internet sources.

Step 3: Constructing Instruments and Validation

A questionnaire for mathematics teachers' perceptions and a test for Grade Eight students' misconceptions in geometry were constructed under the guidance of the supervisor. After preparing the instruments, in order to get validation, expert review was conducted by three experts in the field of mathematics from Mathematics Department, four experts in the field of mathematics from the Department of Methodology and two experts who had experience in teaching mathematics from the basic education middle schools and high schools. Since ambiguities were found in the responses, a little change was made in the original questionnaire after consulting experts.

Step 4: Pilot Testing

The questionnaire for mathematics teachers' perceptions and the test for Grade Eight students' misconceptions in geometry were validated through pilot testing with four middle school mathematics teachers and fifty Grade Eight students. The pilot testing for the instruments was conducted in B.E.H.S 2 (Hmawbi). The internal consistency of the questionnaire for mathematics teachers' perceptions was (.866) and the test for Grade Eight students' misconceptions in geometry was (.740) by using Cronbach's Alpha.

Step 5: Sampling

After the pilot test, the sample schools for this study were selected by using stratified random sampling method. Two high schools and one middle school were selected from each district. Therefore, eight high schools and four middle schools were included in this study. There were (68) middle school mathematics teachers and (600) Grade Eight students participated in this study.

Step 6: Data Collection

The modified instruments were distributed to all participants of the twelve sample schools with the help of the headmaster/headmistress of those schools in December 2018. After that, students' answer sheets were scored manually based on the marking scheme.

Step 7: Analysis of the Data

All the data were organized into the computer data file and were analyzed using the Statistical Package for the Social Science (SPSS 22).

Instruments

In this study, a questionnaire for mathematics teachers' perceptions and a test for Grade Eight students' misconceptions in geometry were used as the instruments. The questionnaire for mathematics teachers' perceptions was mainly based on questionnaire developed by Wong et al. (2001) and questionnaire developed by Kakai (2012). It included causes of students' misconceptions in geometry. It consisted of (15) items on a five point Likert scale of (1) to (5) to explore attitude, attention, participation, doing exercises and conceptual understanding. A test for Grade Eight students' misconceptions in geometry was mainly based on three-tier diagnostic test

developed by Ningrum et al. (2018) and three-tier diagnostic test developed by Jauhariyah et al. (2018). It consisted of three levels of questions: the first level contained the usual multiple-choice questions, the second level contained the choice of reason, and the third level contained questions relating to the belief in the answer chosen in two the previous level. Using this test, it can be known the students who understand the concept, students who have experience misconception, and students who do not understand the concept (see Appendix).

Population and Sample

All participants in the sample were Grade Eight students and middle school mathematics teachers. This study was conducted in Yangon Region. There are four districts in Yangon Region. One township from each district was randomly selected for this study. The sample schools for the study were selected by using a stratified random sampling technique. Two high schools and one middle school from each township were selected as the sample schools. Therefore, eight high schools and four middle schools are included in this study. Grade Eight students and middle school mathematics teachers from selected schools were selected as the sample. The number of students was (600) and the number of teachers was (68).The participants in this study were selected by using a random sampling method and a stratified random sampling technique respectively.

Data Analysis

The data were analyzed by using descriptive statistics (mean, standard deviation and percentage). In order to know the students' misconceptions in geometry, their achievement and their confidence in performing geometric problems, mean, standard deviation and percentage were used. In addition, the responses to the questionnaire for mathematics teachers' perceptions were analyzed to investigate the mathematics teachers' perceptions on the causes of students' misconceptions in geometry.

Findings

Findings of Students' Misconceptions in Geometry

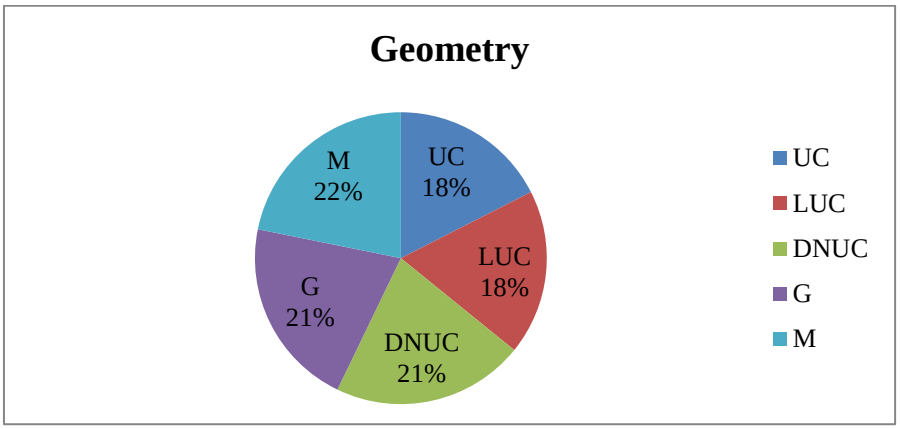

Figure 1: Percentage of Students' Responses in Geometry

Note: UC = Understand the Concept LUC = Less Understand the Concept
DNUC = Do Not Understand the Concept G = Guessing
M = Misconception

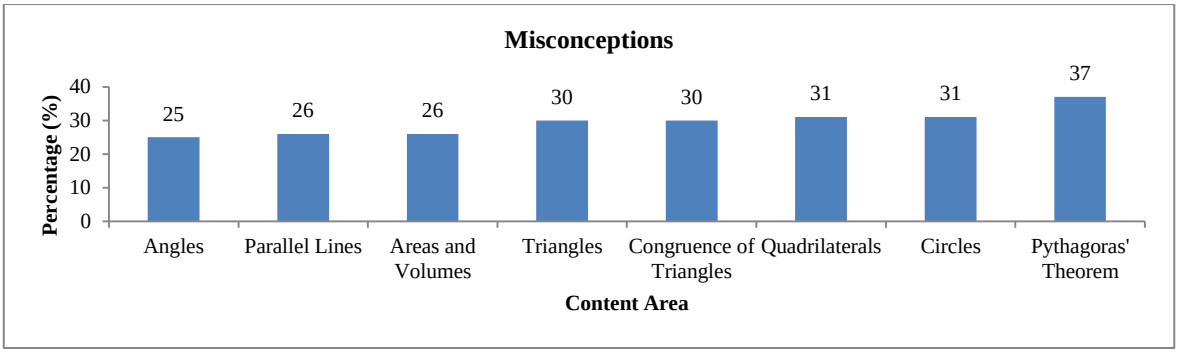

Figure 2 Comparison of Percentage of Students' Misconceptions in Content Areas of Geometry

Findings of Mathematics Teachers' Perceptions on the Causes of Students' Misconceptions in Geometry

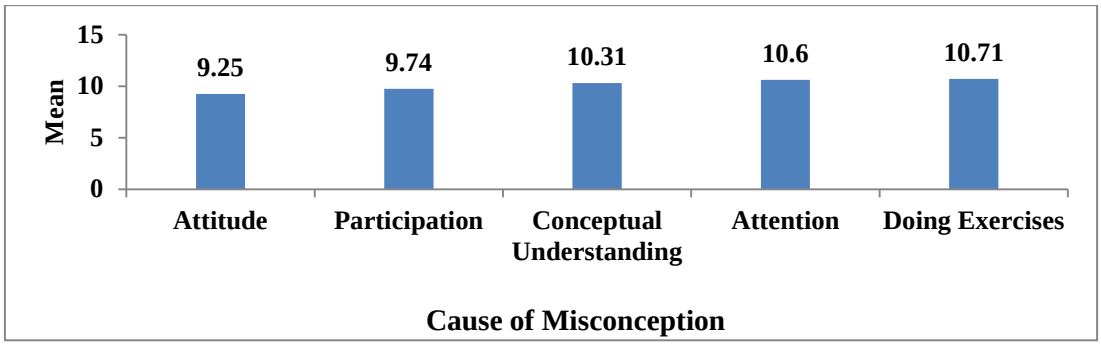

Figure 3 Comparison of Mean for Causes of Students' Misconceptions in Geometry Summary of Findings

To sum up, the findings can be generalized as follows:

- According to a test for Grade Eight students' misconceptions in geometry, Grade Eight students have 22% misconceptions in geometry.
- Grade Eight students have 25% misconceptions in angles.
- Grade Eight students have 30% misconceptions in triangles and congruence of triangles respectively.
- Grade Eight students have 31% misconceptions in quadrilaterals and circles respectively.
- Grade Eight students have 26% misconceptions in parallel lines and areas and volumes respectively.
- Grade Eight students have 37% misconceptions in Pythagoras' theorem.
- Therefore, for Grade Eight students, the highest level of misconceptions in geometry is in Pythagoras' theorem and the lowest level is in angles.
- According to questionnaire for mathematics teachers' perceptions, the mean of doing exercises (10.71) was the highest among five causes of students' misconceptions in geometry and that of attitude (9.25) was the lowest.
- Thus, for Grade Eight students, the highest level of the causes of misconceptions in geometry is in doing exercises and the lowest level in attitude.

Conclusion

Discussion, suggestions and conclusion will be presented.

Discussion

The main purpose of this study is to study the misconceptions about geometry in middle school learners. The specific purposes are to find out students' misconceptions in geometry, to investigate the mathematics teachers' perceptions on the causes of students' misconceptions in geometry and to give suggestions for reducing misconceptions in geometry at the middle school level. To implement these three purposes, this study was conducted by five research questions. In this part, the findings of the research study will be discussed according to the research questions.

Research Question 1: To what extent do Grade Eight students have misconceptions in geometry?

According to the research findings, concerning the students' responses in geometry (see Figure 1), the researcher found that eighteen percent of students' responses in geometry were Understand the Concept (UC) and Less Understand the Concept (LUC) respectively. Twenty one percent of students' responses in geometry were Do Not Understand the Concept (DNUC) and Guessing (G) respectively. Twenty two percent of students' responses in geometry were Misconception (M). Therefore, most of the students have misconceptions in geometry and least of the students have understanding the concept and less understanding the concept in geometry.

Research Question 2: In which content area do Grade Eight students have the highest level of misconceptions in geometry?

Research Question 3: In which content area do Grade Eight students have the lowest level of misconceptions in geometry?

Then, the research questions 2 and 3 will be discussed. Figure 2 was illustrated by arranging the percentages of misconception in content areas of geometry in ascending order. According to Figure 2, the first is the percentage of misconception in angles. The second is the percentage of misconception in parallel lines and areas and volumes. The fourth is the percentage of misconception in triangles and congruence of triangles. The sixth is the percentage of misconception in quadrilaterals and circles. The last is the percentage of misconception in Pythagoras' theorem. Therefore, for Grade Eight students, the highest level of misconceptions in geometry is in Pythagoras' theorem and the lowest level is in angles.

Research Question 4: In which cause do Grade Eight students have the highest level of misconceptions in geometry?

Research Question 5: In which cause do Grade Eight students have the lowest level of misconceptions in geometry?

Besides, the research questions 4 and 5 will be discussed. Figure 3 was illustrated by arranging the means for the causes of students' misconceptions in geometry in ascending order. According to Figure 3, the first is the mean of attitude. The second is the mean of participation. The third is the mean of conceptual understanding. The fourth is the mean of attention and the fifth is the mean of doing exercises. Therefore, for Grade Eight students, the highest level of the causes of misconceptions in geometry is in doing exercises and the lowest level is in attitude.

According to the results of the above presentation, the highest level of the causes of misconceptions in geometry was in doing exercises (rote learning). This result implies that most of the students are learning by rote. Rote learning can make students to be inadequate thinking and reasoning abilities. Thus, it is necessary to reduce rote learning.

According to National Education Strategic Plan (NESP) (2016), the current emphasizes on rote memorization of factual information in learning and assessment. Although there are many advantages of rote learning, rote learning can lead to many disadvantages. Clements and Battista (1992, cited in Pusey & Lousie, 2003) indicated the reasons for learners' misconceptions about geometric concepts as follows: learners do not understand subjects sufficiently, they overgeneralize specific rules about geometric expressions, they mostly learn by rote, and they cannot understand concepts exactly.

According to the research findings, most of the students have misconceptions in geometry, least of the students have understanding the concept and less understanding the concept in geometry, and the highest level of the causes of misconceptions in geometry is in doing exercises (rote learning). These results are consistent with the facts indicated by Clements and Battista (1992, cited in Pusey & Lousie, 2003). Therefore, to reduce misconceptions, it is necessary to ensure meaningful learning and quality education.

Suggestions

To reduce learners' misconceptions in geometry and rote learning, it is necessary to ensure meaningful learning and quality education. To become meaningful learning and quality education, effective teaching and learning is essential. That is why; teachers should use appropriate methods according to the level of geometric thinking of students. Teachers should use three stages of teaching geometry: the practical stage, the stage of reasoning and the systematizing stage. If students do not have the underlying foundation of what actually means, students will struggle with learning geometry. Therefore, teachers should not be concentrated on facts. Learning by doing can make students raise the level of recall and retention of the content in long-term memory. Thus, teaching should be done in using visual aids, manipulatives, games and puzzles, etc.

Besides, students should learn geometry by small group cooperative learning. Small group provides a forum in which students ask questions, discuss ideas, demonstrate to others, learn to listen to others and offer constructive criticism and summarize their discoveries in writing. If so, students can reduce anxiety and misconceptions. Furthermore, to become effective teaching and learning, the nature of the classroom should be suitable for actively oriented pedagogy. This classroom should provide learners to interact with teacher without fear and move freely in it. It should also provide opportunity for variety of activity. Sufficient learning materials should available in this classroom. There should be freedom of expression and need of the learners should be taken into consideration. Areas like reference corner, reading corner, information corner, classroom library, etc. should set up in the classroom.

These are some suggestions to reduce misconceptions and ensure meaningful learning and quality education. Moreover, many factors to take into account before learning and after learning are also remaining in order to become meaningful learning. In the teaching and learning process, the teacher is essential. Thus, to become effective teaching and learning, teachers should

continuously try to know the factors that support meaningful learning and to upgrade their pedagogical content knowledge.

Conclusion

In the 21st century, learning is an active process in which learners construct new ideas or concepts based upon their current and past knowledge. If learners have actively constructed misconceptions, they will think everything according to misconceptions and solve various problems by using them. They will definitely interfere with learning. Therefore, it is necessary to reduce misconceptions to become effective learning.

Besides, one of the challenges for mathematics educators is not only enabling their charges to know more and be able to do more with their mathematics, but to have a greater affinity for mathematics itself (Allen, 2011, cited in Foley, 2016). Therefore, teachers should try students to have positive attitudes towards mathematics.

Positive student perceptions of mathematics can link to higher student achievement. These can contribute to students in their daily lives. If students have misconceptions in geometry, these can lead to confusion, frustration, errors and prevent learners to build up confidence and positive attitude towards learning, to value mathematics and to appreciate the beauty of mathematics. Therefore, it is necessary to reduce students' misconceptions. To reduce misconceptions in geometry, it is also necessary to understand why these misconceptions emerge and persist.

According to the research findings, most of the students had misconceptions in geometry, least of the students had understanding the concept and less understanding the concept in geometry, and the highest level of the causes of students' misconceptions in geometry was in doing exercises (rote learning). Therefore, it is necessary to ensure meaningful learning and quality education.

To become meaningful learning and quality education, teaching and learning should be effective. To become effective teaching and learning, teachers should know the previous knowledge of their students and use appropriate methods according to the level of students' geometric thinking. Teachers should teach students by using visual aids, manipulatives, games and puzzles, etc. Furthermore, students should learn small group cooperative learning. To be convenience in learning geometry, a proper atmosphere should be created in the classroom.

Furthermore, teachers should use not only summative assessment but also formative assessment. Assessment should not emphasize on rote memorization of factual information. It should emphasize students' thinking and reasoning skills. If teachers find out students' misconceptions according to the result of assessment, teachers should not ignore these misconceptions. Teachers should provide remediation by using appropriate methods. Only when, teachers will reduce students' misconceptions.

Although this study cannot fulfill the all objectives of teaching mathematics at the middle school level in Myanmar, it can be a support for teachers and curriculum planners to understand the middle school learners' misconceptions and address conceptual difficulties.

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Appendix

အောက်ပါ မေးခွန်းများကို ဖတ်ရှု၍ ပေးထားသော ဖော်ပြချက်များမှ မှန်ကန်သော အဖြေကို စက်ဝိုင်းဝိုင်းပေးပါ။

ဥပမာ

1.	a.	အောက်ပါ ထောင့်တွဲများအနက် ထောင့်မှန်ဖြည့်ဖက် အတွဲမှာ (A.) 30°, 60° B. 40°, 140° C. 70°, 200° D. 90°, 270° ဖြစ်သည်။
	b.	ထောင့်နှစ်ထောင့်သည် ထောင့်မှန်ဖြည့်ဖက် ဖြစ်လျှင် ၄င်းထောင့်နှစ်ထောင့် ပေါင်းလဒ်သည် (A.) 90° B. 180° C. 270° D. 360° ရှိသည်။
	c.	သင်ဖြေဆိုထားသည့် အဖြေနှင့် ပတ်သက်၍ (i) သေချာပါသည်။ (ii) မသေချာပါ။

Marking Scheme

No. of Items = 22 items
Each question = 1 mark
သေချာပါသည် = 1 မသေချာပါ = 0

Each Item			**Problem Level Analysis**
a	**b**	**c**	**Category**
1	1	1	Understand the Concept
1	1	0	Less Understand the Concept
0	0	0	Do Not Understand the Concept
1	0	0	Guessing
0	1	0	
1	0	1	Misconception
0	0	1	
0	1	1	